1. Which of the following sets are subspace of the vector space C(a, b) of all real valued continuous function defined on the open interval (a, b)?

 (a) {f ∈ C(a, b) : f(x0) = 0, x0 ∈ (a, b)} (b) {f ∈ C(a, b) : f’(x) ≠ 0, for all x ∈ (a, b)}

 (c)  (d) None of these

**Ans. (a)**

**Explanation :**

 (a) Let f ∈ C(a, b) & g ∈ C(a, b)

 Then (αf + βg)(x0) = αf(x0) + βg(x0) = 0

 So, αf + βg ∈ C(a, b)

 Option (a) is correct.

 (b) Let W = {f ∈ C(a, b) : f’(x) ≠ 0 for all x ∈ (a, b)}

 Hence f(x) = 0 ∉ W

 Option (b) is incorrect.

 (c) Let 

 Hence f(x) = 0 ∉ W

 Option (c) is incorrect.

 (d) Option (d) is incorrect.

2. Let T : R2 → R3 be a linear transformation with T(1, 1) = (0, 0, 1)

 T(1, 2) = (0, 1, 1)

 Then T(5, 2) is

 (a) (0, 0, 1) (b) (0, -3, 5) (c) (0, 3, 5) (d) (0, 5, 3)

**Ans. (b)**

**Explanation :**

 Let (5, 2) ∈ R2

 Then (5, 2) = α(1, 1) + β(1, 2)

 (5, 2) = (α + β, α + 2β)

 ⇒ α + β = 5 & α + 2β = 2

 So, α + β + β = 2

 ⇒ β = 2 – 5 = -3 & α = 8

 So, T(5, 2) = 8T(1, 1) - 3T(1, 2)

 = (8)(0, 0, 1) + (-3) (0, 1, 1)

 So, T(5, 2) = (0, -3, 5)

 Option (b) is correct.

3. Let T : R3 → R3 be the linear operator defined by T(X) = AX, where 

 Then the range and kernel of T are

 (a) A line and a plane not passing through origin

 (b) A line & a plane passing through origin

 (c) A plane & a line passing through origin

 (d) Two plane passing through origin

**Ans. (c)**

**Explanation :**

 We know that ρ(T) = ρ(A) & η(T) = η(A)

 Given that 

Here η(A) = 1 & ρ(A) = 2

So, Option (c) is correct.

4. The linear transformation T : R3 → R2 defined by T(x, y, z) = (x + y, y + z) is

 (a) Linear and has zero kernel (b) Linear & has a proper kernel as subspace

 (c) Neither linear nor onto (d) None of these

**Ans. (b)**

**Explanation :**

 ker(T) = {(x, y, z) ∈ R3 | T(x, y, z) = 0}

 ker(T) = {(x, y, z) ∈ R3 | x + y = 0, y + z = 0}

 ker T = <1, -1, 1>

 dim ker(T) = 1

 Option (b) is correct.

5. Which of the following sets of vectors in R3 are LI

 I. {(1, 0, 0), (0, 1, 0), (1, 1, 0)}

 II. {(1, 0, 0), (0, 1, 0), (0, 0, 1)}

 III. {(0, 1, 0), (0, 0, 1), (0, 1, 1)}

 Select the correct statement?

 (a) I and II (b) II only (c) III only (d) II & III

**Ans. (b)**

**Explanation :**

I. {(1, 0, 0), (0, 1, 0), (1, 1, 0)}

 Here (1, 1, 0) = (1, 0, 0) + (0, 1, 0)

 III. {(0, 1, 0), (0, 0, 1), (0, 1, 1)}

 Here (0, 1, 1) = (0, 1, 0) + (0, 0, 1)

 II. is true.

6. Let W1 & W2 are finite dimensional vector subspace V. If dim W1 = 2, dim W2 = 2, dim (W1 + W2) = 3, then dim (W1 ∩ W2) is

 (a) 1 (b) 2 (c) 3 (d) 4

**Ans. (a)**

**Explanation :**

We know that

 dim (W1 ∩ W2) = dim W1 + dim W2 – dim (W1 ∩ W2)

 = 2 + 2 – 3

 = 1

7. Let (α, β, γ) be the coordinate of vector of (3, 5, -2) relative to the standard basis {(1, 0, 0), (0, 1, 0), (0, 0, 2)} then (α, β, γ) is

 (a) (2, 3, 5) (b) (3, 5, -2) (c) (3, 5, 2) (d) (3, 5, -1)

**Ans. (d)**

**Explanation :**

(3, 5, -2) = α(1, 0, 0) + β(0, 1, 0) + γ(0, 0, 2)

 = (α, β, 2γ)

 = α = 3, β = 5, γ = -1

8. Which of the following are linear transformation from R3 to R2?

 (a) T(x, y, z) = (x, y, z) (b) T(x, y, z) = (xy, z)

 (c) T(x, y, z) = (|x|, 0) (d) T(x, y, z) = (x2, 0)

**Ans. (a)**

**Explanation :**

(b) Given that T(x, y, z) = (x, y, z)

 Which is not linear transformation.

 Option (b) is incorrect.

 (c) T(x, y, z) = (|x|, 0)

 |x| is not in linear form.

 Option (c) is incorrect.

 (d) T(x, y, z) = (x2, 0)

 x2 is not in linear form.

 Option (d) is incorrect.

1. Option (a) is correct.

9. Let T be the linear operator on R3 defined by T(x1, x2, x3) = (2x1, x1 – x2, 5x1 + 4x2 + x3). Then T-1 (0, 1, 1)

 (a) (1, 5, 0) (b) (0, -1, 5) (c) (5, 0, 1) (d) (0, 5, -1)

**Ans. (b)**

**Explanation :**

Let T(x1, x2, x3) = (a, b, c)

 Then T-1(a, b, c) = (x1, x2, x3)

 Now for T(x1, x2, x3) = (a, b, c)

 (2x1, x1 – x2, 5x1 + 4x2 + x3) = (a, b, c)

 x1 = a/2, x1 – x2 = b, 5x1 + 4x2 + x3 = C

 







So, T-1(0, 1, 1) = (0, -1, 5)

Option (b) is correct.

10. Let V be vector space of dimension 3 over R. Let T : V → V be a linear transformation given by matrix  w.r.t. a order basis {V1, V2, V3} of V. Then which of the following are true?

 (a) T(V3) = 0 (b) T(V1 + V2) = 0 (c) T(V1 + V2 + V3) = 0

 (d) None of these

Ans. (c)

Explanation :

 Given that 

 T(V1) = V1 + V2 – 2V3

 T(V2) = -V1 – 4V2 + 5V3

 T(V3) = 3V2 – 3V3

 (a) T(V3) ≠ 0

 Option (a) is incorrect.

 (b) T(V1) + T(V2) = V1 + V2 – 2V3 – V1 – 4V2 + 5V3

 = 3V3 – 3V2 ≠ 0

 Option (b) is incorrect.

 (c) T(V1) + T(V2) + T(V3)

 = V1 + V2 – 2V3 – V1 – 4V2 + 5V3 + 3V2 – 3V3

 = 0

 Option (c) is correct.

11. The dimension of the vector space of all symmetric matrices of order 3 × 3 with real entries and Trace is zero

 (a) 4 (b) 5 (c) 9 (d) 6

**Ans. (b)**

12. Let V be a 3-dimensional vector space over the field Z3 of 3 elements. The number of distinct 2-dimensional subspace of V is

 (a) 13 (b) 17 (c) 15 (d) 14

**Ans. (a)**

13. Which of the following sets of functions from R to R is a vector space over R

  

 (a) S1 but not S2 (b) S2 but not S1 (c) S1 is not (d) neither S1 nor S2

**Ans. (c), (d)**

14. Let V be a vector space and W is subspace of V, then which of the following property hold?

 (a) , for all V

 (b) , when V is finite dimensional vector space.

 (c) Number of L.I. condition in W.

 (d) None of these

**Ans. (b), (c)**

**Explanation :**

If V is finite dimensional vector space, then but it is not true

 for infinitely dimensional vector space.

 Option (a) is not true, option (b) is true.

 Option (c) is always true.

15. Consider the following subspace of R3.

 W = {(x, y, z) ∈ R3 | 2x + 2y + z = 0, x + y – 3z = 0}. Then dim(W) is equal to dimension of

 (a) R(R) (b) R2(R) (c) R4 (R) (d) R3(R)

**Ans. (a)**

**Explanation :**



Both restriction are LI

 So, dim W = 3 – 2 = 1

16. Which of the following are not subspace of R3?

 (a) {(x, y, z) ∈ R3 | x + 2y = 0, 2y + 2z = 0}

 (b) {(x, y, z) ∈ R3 | x + y + 3z – 3 = 0}

 (c) {(x, y, z) ∈ R3 | x – 1 = 0, z = 0}

 (d) {(x, y, z) ∈ R3 | x = y = 0}

**Ans. (b), (c)**

**Explanation :**

(b) (0, 0, 0) ∉ W

 (c) (0, 0, 0) ∉ W

 Option (b) and (c) are not subspace of R3.

17. Let T : R3 → R3 be the linear transformation defined by T(x, y, z) = (x + y, y + z, z + x), then which of the following is/are true?

 (a) A is one-one & onto (b) ρ(T) = 3 & η(T) = 1

 (c) ρ(T) = 2 & η(T) = 1 (d) ρ(T) = 3 & η(T) = 0

**Ans. (a), (d)**

**Explanation :**

T(1, 0, 0) = (1, 0, 1)

 T(0, 1, 0) = (1, 1, 0)

 T(0, 0, 1) = (0, 1, 1)

 

 Here ρ(A) = 3 & η(A) = 0

18. Let V be the vector space of all polynomial with real coefficient of degree atmost 3 and the linear transformation from V to V s.t. .

 Let A = {[aij]4×4; aij ∈ R} be a matrix of linear transformation then a12 + a21 is

 (a) 1 (b) 1/2 (c) 2 (d) 3/2

**Ans. (b)**

**Explanation :**

Basis of V is β = {1, x, x2, x3}

 

 Now 

 a21 = coefficient of x in T(1) ⇒ a21 = 0

 Now, 

 

 

**19.** Let V be the real vector space of all polynomial of degree ≤3 with real coefficients. Define the linear transformation T(α0 + α1x + α2x2 + α3x3)

 = α0 + α1(x + 1) + α2(x + 1)2 + α3(x + 1)3

 Then the rank of the matrix of T with respect to the basis {1, x, x2, x3} of V

 (a) 1

 (b) 2

 (c) 3

 (d) 4

**Ans. (d)**

**Explanation :**

Given that a basis β = {1, x, x2, x3}

 Then T(1) = T(1 + 0.x + 0.x2 + 0.x3) = 1

 T(x) = (x + 1)

 T(x2) = (x + 1)2 = x2 + 2x + 1

 T(x3) = 1 + 3x + 3x2 + x3

 Then matrix representation

 T(1) T(x) T(x2) T(x3)

 

 Then ρ(A) = 4

20. Let p(x) = {a0 + a1x + a2x2 + a3x3 | ai∈R, i = 0, 1, 2, 3}. under the standard operation of addition (+) and scalar multiplication ( . ) then p(x) is

 (a) Not a vector space

 (b) A vector space of infinite dimension

 (c) A vector space of dimension 3

 (d) A vector space of dimension 4

**Ans. (d)**

**Explanation :**

Given that p(x) is the set of all polynomial of degree at most 3

 ⇒ p(x) = R3[x]

 Then p(x) is vector space of dimension 4.